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The influence of surface mass flux on mixed convection over horizontal plates in saturated porous media

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INTRODUCTION

THE FIRST attempts to study mixed convection in a porous layer were those of Wooding [1], Prats [2], Sutton [3], and Homsy and Sherwood [4]. The emphases in these studies, however, were on the stability of the flow field and the establishment of the criterion for the onset of convection. Experimental results were very limited and were reported only by Combarnous and Bia [5]. By using a boundary-layer formulation and the similarity method, Cheng [6, 7] conducted a series of investigations to study mixed convection over vertical, inclined and horizontal plates in porous media. Recently, numerical results of mixed convection in vertical and horizontal porous layers with non-uniform heating on the boundary have been reported [8, 9], while experimental results have been reported only for the latter case [10, 11].

As a continuing effort toward a complete understanding of transport phenomena in porous media, we consider in this note the influence of surface mass transfer on mixed convection over horizontal plates in saturated porous media. The approach follows that used by Cheng and co-workers [12, 13] for the study of free convection. Similarity solutions are obtained for the special case where the surface temperature, free stream velocity and injection, or withdrawal, velocity are a prescribed power function of distance. The limiting cases of free and forced convection are also examined.

ANALYSIS

Consider the problem of injection or withdrawal of fluid through the surface of a horizontal plate embedded in a saturated porous medium. Having invoked the Boussinesq and boundary-layer approximations, the governing equations based on Darcy's law are given by

$$\frac{\partial^2 \Psi}{\partial y^2} = -\frac{Kg\beta}{\nu}\frac{\partial T}{\partial x} \quad (1)$$

$$\frac{\partial \Psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \Psi}{\partial x}\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (2)$$

with boundary conditions

$$y = 0, \quad T_w = T_\infty + Ax^\lambda, \quad v = -\frac{\partial \Psi}{\partial x} = v_w = ax^n \quad (3)$$

$$y \to \infty, \quad T = T_\infty,$$

$$u = \frac{\partial \Psi}{\partial y} = 0, \text{ for free convection} \quad (4a)$$

$$= U_\infty = BX^m, \text{ for mixed convection} \quad (4b)$$

and a is positive for injection of fluid and negative for withdrawal of fluid. With the properly chosen similarity variables, equations (1) and (2) can be transformed to a set of ordinary differential equations.

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Free and mixed convection

The suitable similarity variables are

$$\eta = (Ra)^{1/3}\frac{y}{x} \quad (5)$$

$$\Psi = \alpha (Ra)^{1/3} f(\eta) \quad (6)$$

and

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}. \quad (7)$$

After transformation, the resulting equations are

$$f'' = -\left(\frac{\lambda - 2}{3}\eta\theta' + \lambda\theta\right) \quad (8)$$

$$\theta'' = \lambda f'\theta - \frac{\lambda + 1}{3} f\theta' \quad (9)$$

with boundary conditions given by

$$\eta = 0, \quad \theta = 1, \quad f = [f_w]_{nc} = C^{-1/3}[f_w]_{mx} \quad (10)$$

$$\eta \to \infty, \quad \theta = 0, \quad f' = C^{2/3} \quad (11)$$

where

$$C = Pe^{3/2}/Ra \quad (12)$$

is the parameter for mixed convection. It is clear that solutions with $C = 0$ correspond to free convection. In equation (10), $[f_w]_{nc}$ is the mass flux parameter for free convection

$$[f_w]_{nc} = -\frac{3a}{\alpha(\lambda+1)\left[\frac{Kg\beta A}{\nu\alpha}\right]^{1/3}} \quad (13)$$

and $[f_w]_{mx}$ is the mass flux parameter for mixed convection

$$[f_w]_{mx} = -\frac{2a}{(\alpha B)^{1/2}(1+m)}. \quad (14)$$

It is clear that f_w is positive for the withdrawal of fluid and negative for injection.

For free convection, it has been shown that similarity solutions exist for the case of an impermeable plate [14], i.e. $[f_w]_{nc} = 0$, no injection or withdrawal of fluid. Similarly, it can be shown that equations (8) and (9) also permit similarity solutions if $n = (\lambda - 2)/3$. However, as pointed out by Cheng and Chang [14], the solutions are physically realistic only when $1/2 \leqslant \lambda \leqslant 2$. For $\lambda = 1/2$, this corresponds to the case of a plate heated with constant flux while for $\lambda = 2$, it corresponds to a constant suction or injection velocity on the surface. For the latter case, Minkowycz *et al.* [13] have presented a non-similarity analysis for a wide range of mass flux parameters and wall temperature distributions. The solutions for the first case, however, have not been reported before.

For mixed convection, it is clear that equations (8) and (9) will permit similarity solutions if $\lambda = (3m+1)/2$ and $n = (m-1)/2$. For $\lambda = 1/2$, this corresponds to the case of a uniform flow over a horizontal plate heated with constant flux. For $\lambda = 2$, it corresponds to a stagnation flow over a horizontal plate with constant suction or injection of fluid

NOMENCLATURE

A	constant defined in equation (3)
a	constant defined in equation (3)
B	constant defined in equation (4b)
C	mixed convection parameter defined by equation (12)
f	dimensionless stream function defined by equations (6) and (16)
f_w	surface mass flux parameter defined by equations (13) and (14)
g	acceleration due to gravity [m s^{-2}]
h	local heat transfer coefficient [W m^{-2} K^{-1}]
K	permeability [m^2]
k	effective thermal conductivity [W m^{-1} K^{-1}]
m	constant defined in equation (4b)
n	constant defined in equation (3)
Nu	local Nusselt number, hx/k
Pe	local Peclet number, $U_\infty x/\alpha$
Ra	modified local Rayleigh number, $Kg\beta(T_w - T_\infty)x/\nu\alpha$
T	temperature [K]
U_∞	free stream velocity in the x-direction [m s^{-1}]
u	Darcy's velocity in the x-direction [m s^{-1}]
v	Darcy's velocity in the y-direction [m s^{-1}]
v_∞	fluid injection or withdrawal velocity [m s^{-1}]
x, y	Cartesian coordinate [m].

Greek symbols

α	thermal diffusivity of porous medium [m^2 s^{-1}]
β	coefficient of thermal expansion [K^{-1}]
η	independent similarity variable
η_T	dimensionless thermal boundary-layer thickness
θ	dimensionless temperature
λ	constant defined in equation (3)
ν	kinematic viscosity of convective fluid [m^2 s^{-1}]
Ψ	stream function.

Subscripts

fc	forced convection
mx	mixed convection
nc	natural convection
w	condition at the wall
∞	condition at infinity.

Table 1. Selected values of $-\theta'(0)$, $f'(0)$ and η_T for free convection over a horizontal plate in a saturated porous medium

f_w	$-\theta'(0)$	$f'(0)$	η_T
		$\lambda = 1/2$	
−0.8	0.6835	1.3044	5.1691
−0.4	0.7433	1.2190	4.9818
0.0	0.8125 (0.8164)†	1.1330	4.7633 (5.0)†
0.2	0.8510	1.0900	4.6386
0.4	0.8923	1.0472	4.5073
0.6	0.9366	1.0049	4.3751
0.8	0.9839	0.9629	4.2253
1.0	1.0344	0.9217	4.0761
		$\lambda = 2$	
−0.8	1.3853 (1.387)‡	1.7062 (1.708)‡	4.5155
−0.4	1.4688 (1.471)	1.5777 (1.579)	4.0574
0.0	1.5702 (1.571)	1.4467 (1.447)	3.6719 (3.7)†
0.2	1.6288 (1.629)	1.3803 (1.380)	3.4708
0.4	1.6920 (1.692)	1.3124 (1.313)	3.2604
0.6	1.7632 (1.763)	1.2453 (1.246)	3.0610
0.8	1.8421 (1.842)	1.1787 (1.179)	2.8702
1.0	1.9298 (1.930)	1.1134 (1.113)	2.6886

† Solutions presented by Minkowycz *et al.* [13].
‡ Solutions presented by Cheng and Chang [14] for an impermeable plate.

on the surface. For flow over an impermeable plate, i.e. $[f_w]_{mx} = 0$, the solutions have also been reported by Cheng [7].

Forced convection

For the limiting case of forced convection, the appropriate similarity variables are

$$\eta = (Pe)^{1/2}\frac{y}{x} \tag{15}$$

$$\Psi = \alpha(Pe)^{1/2}f(\eta). \tag{16}$$

Equations (1) and (2) are transformed to

$$f' = 1 \tag{17}$$

$$\theta'' = \lambda\theta - \frac{m+1}{2}f\theta'. \tag{18}$$

RESULTS AND DISCUSSION

The transformed ordinary differential equations, with the corresponding boundary conditions, are solved by numerical integration using the fourth-order Runge–Kutta method and the shooting technique with a systematic guessing of $\theta'(0)$ and $f'(0)$. Selective values of $-\theta'(0)$ and $f'(0)$ are listed in Table 1 for free convection, and in Table 2 for mixed convection. As an indication of proper formulation and accurate calculation, the results thus obtained have been compared with the data published earlier, and they show excellent agreement.

The heat transfer coefficient in terms of Nusselt number is given by

$$\frac{Nu}{Ra^{1/3}} = [-\theta'(0)]_{nc} \text{ for free convection} \tag{19}$$

Table 2. Selected values of $-\theta'(0)$, $f'(0)$ and η_T for mixed convection over a horizontal plate in a saturated porous medium

	$f_w = -1$			$f_w = 0$			$f_w = 1$		
$Ra/Pe^{3/2}$	$-\theta'(0)$	$f'(0)$	η_T	$-\theta'(0)$	$f'(0)$	η_T	$-\theta'(0)$	$f'(0)$	η_T
					$\lambda = 1/2$				
0 (fc)	0.6337	1.0	3.8245	0.8862 (0.8862)†	1.0 (1.000)†	3.2153 (3.2)†	1.2009	1.0	2.6934
0.6	0.8037	1.5728	3.3938	1.0281 (1.028)	1.4740 (1.474)	2.9486 (2.9)	1.3123	1.3907	2.5428
1.0	0.8862	1.8862	3.2154	1.1020 (1.102)	1.7474 (1.747)	2.8250 (2.8)	1.3745	1.6264	2.4624
2.0	1.0450	2.5547	2.9088	1.2495 (1.249)	2.3479 (2.348)	2.6049 (2.6)	1.5041	2.1591	2.3115
5.0	1.3575	4.1212	2.4379	1.5503 (1.550)	3.7996 (3.799)	2.2380 (2.2)	1.7825	3.4927	2.0347
8.0	1.5724	5.3921	2.1953	1.7610 (1.761)	4.9990 (4.999)	2.0340 (2.0)	1.9836	4.6181	1.8688
15.0	1.9290	7.8493	1.8704	2.1137 (2.113)	7.3444 (7.345)	1.7535 (1.7)	2.3261	6.8498	1.6377
					$\lambda = 2$				
0 (fc)	1.1258	1.0	2.5646	1.5957 (1.595)†	1.0 (1.000)†	2.0472 (2.0)†	2.2117	1.0	1.6378
0.6	1.4652	1.7240	2.3195	1.8627 (1.863)	1.5778 (1.578)	1.9175 (1.9)	2.4064	1.4579	1.5730
1.0	1.6309	2.1235	2.2138	2.0044 (2.004)	1.9159 (1.916)	1.8532 (1.8)	2.5182	1.7391	1.5371
2.0	1.9494	2.9781	2.0284	2.2889 (2.291)	2.6630 (2.666)	1.7334 (1.7)	2.7574	2.3852	1.4656
5.0	2.5716	4.9815	1.7304	2.8820 (2.879)	4.4981 (4.495)	1.5188 (1.5)	3.2827	4.0268	1.3230
8.0	2.9971	6.6069	1.5729	3.2927 (3.292)	6.0105 (6.010)	1.3954 (1.3)	3.6683	5.4273	1.2327
15.0	3.6971	9.7429	1.3504	3.9742 (3.982)	8.9644 (8.980)	1.2306 (1.2)	4.3308	8.2232	1.1000

† Solutions presented by Cheng [7] for impermeable surfaces in saturated porous media.

$$\frac{Nu}{Pe^{1/2}} = [-\theta'(0)]_{mx} \text{ for mixed convection} \quad (20a)$$

$$= [-\theta'(0)]_{fc} \text{ for forced convection.} \quad (20b)$$

For free convection, it is clearly observed that the introduction of surface mass transfer has a significant influence on the heat transfer results (Fig. 1). While the withdrawal of fluid from the surface has greatly enhanced the heat transfer rate, the injection of fluid has considerably decreased it.

For mixed convection, the heat transfer result is plotted in Fig. 2 as a function of f_w and $Ra/Pe^{3/2}$. The limiting cases of free and forced convection are also shown as asymptotes in the same figure. The influence of surface mass flux on mixed convection is clearly observed from these figures. As is the case for free convection, the heat transfer rate increases with the mass flux parameter, f_w, i.e. enhancement for suction and reduction for injection.

For the case of impermeable surfaces, i.e. $[f_w]_{mx} = 0$, the free convection asymptotes are linear and are given by

$$\frac{Nu}{Pe^{1/2}} = 0.8125\left[\frac{Ra}{Pe^{3/2}}\right]^{1/3} \quad \text{for } \lambda = 1/2$$

$$= 1.5702\left[\frac{Ra}{Pe^{3/2}}\right]^{1/3} \quad \text{for } \lambda = 2. \quad (21)$$

For $[f_w]_{mx} \neq 0$, the corresponding free convection asymptotes can be obtained by rewriting equation (20) as

$$\frac{Nu}{Pe^{1/2}} = \frac{Nu}{Ra^{1/3}}\left(\frac{Ra}{Pe^{3/2}}\right)^{1/3} = C^{-1/3}[-\theta'(0)]_{nc} \quad (22)$$

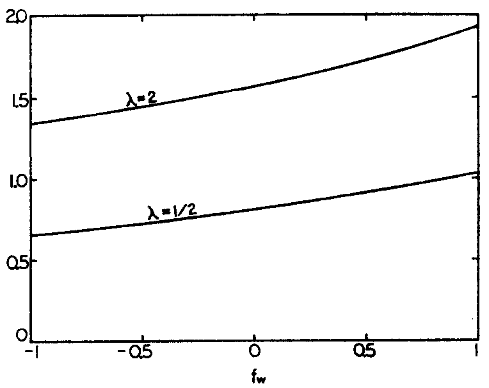

FIG. 1. Effects of surface mass flux on heat transfer results for free convection over a horizontal plate in a saturated porous medium.

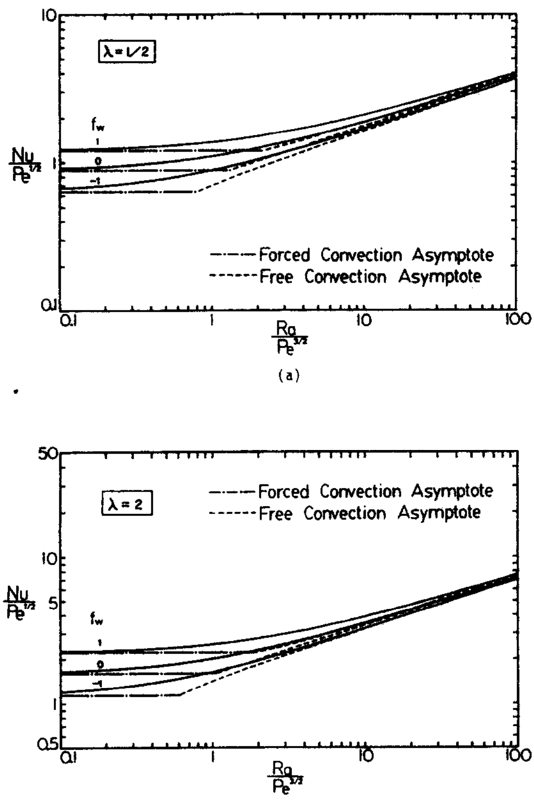

FIG. 2. Effects of surface mass flux on heat transfer results for mixed convection over a horizontal plate in a saturated porous medium.

and applying the relation between $[f_w]_{nc}$ and $[f_w]_{mx}$ as given by equation (10).

With a given $[f_w]_{mx}$ and C, $[f_w]_{nc}$ can be determined through equation (10). Once $[f_w]_{nc}$ is specified, $[-\theta'(0)]_{nc}$ can be solved from equations (8) and (9). Therefore, the free convection asymptote is obtained, from equation (22), for each corresponding $[f_w]_{mx}$.

To summarize, the influence of surface mass transfer on mixed convection over horizontal plates in saturated porous media has been studied analytically. Similarity solutions have been reported for the special cases for which the wall temperature, free stream velocity and injection or withdrawal velocity are a prescribed power function of distance. It is found that the heat transfer, in the form of free, mixed or forced convection, is enhanced by the withdrawal of fluid from the surface while it is decreased by the injection of fluid. Problems of this kind may be encountered frequently in the geophysical and geothermal applications. Solutions thus obtained, although applicable only to the injection and withdrawal of the same species, provide useful information when surface mass transfer due to chemical reactions has to be considered.

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Evaluation of spherical harmonics approximation for radiative transfer in cylindrical furnaces

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1. INTRODUCTION

IN A PREVIOUS paper [1], the accuracy of several radiation models for three-dimensional radiative heat transfer have been assessed by applying these models to the prediction of the distributions of radiative flux density and the radiative energy source term of a rectangular enclosure problem and by comparing these predictions with exact solutions [2].

A significant number of industrial furnaces and combustors are cylindrical in shape. Therefore, it is considered necessary to evaluate the radiation models produced earlier for cylindrical furnaces by applying them to the prediction of radiative flux density and source term distributions of a cylindrical enclosure problem based on data reported previously on a pilot-scale experimental furnace [3] and by comparing their predictions with exact values reported previously [4].

The radiation model to be tested is the spherical harmonics approximation derived for an axisymmetrical radiation field [5]. In this model the angular variation of intensity at a point is expressed by a series of spherical harmonics. By using the P_1 approximation (in which the series is truncated after the first four terms) and the equation of radiative transfer, the axisymmetrical radiation field within a grey, non-scattering medium is represented by three partial differential equations in the total incident flux density and the net radiant flux densities in the positive coordinate directions [6].

This model had previously been reduced to two-flux form and applied to the prediction of the behaviour of large-scale experimental furnaces and predicted temperature and radiative flux density distributions had been compared with experimentally determined data [6, 7]. However, it has been found impossible to decide whether discrepancies between predictions and measurements are attributable directly to the radiation model employed or to inaccuracies in the submodels used for the prediction of flow, reaction, etc.

The use of exact solutions for testing purposes provides a means for assessing the accuracy of predictions of a radiation model in isolation from the models of flow and reaction.

In this paper, therefore, the accuracy of the P_1 spherical harmonics approximation is tested by applying it to the prediction of the distributions of the radiative flux density